Towards a standard procedure for the measurement of the multi-photon component in a CW telecom heralded single-photon source

To cite this article: E Rebufello et al 2019 Metrologia 56 025004

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Towards a standard procedure for the measurement of the multi-photon component in a CW telecom heralded single-photon source

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Received 19 September 2018, revised 20 December 2018
Accepted for publication 28 January 2019
Published 22 February 2019

Abstract

Single-photon sources are set to be a fundamental tool for metrological applications as well as for quantum information related technologies. Because of their upcoming widespread dissemination, the need for their characterization and standardization is becoming of the utmost importance. Here, we illustrate a strategy to provide a quantitative estimate of the multi-photon component of a single-photon source, showing the results achieved in a pilot study for the measurement of the second-order autocorrelation function $g^{(2)}(0)$ of a low-noise CW heralded single photon source prototype (operating at telecom wavelength $\lambda = 1550$ nm) realized in INRiM. The results of this pilot study, involving INRiM, NPL and PTB, will help to build a robust and unambiguous procedure for the characterization of the emission of a single-photon source.

Keywords: single-photon sources, quantum technologies, quantum characterization

(Some figures may appear in colour only in the online journal)
practice, blurs their deterministic behavior with respect to intrinsically probabilistic heralded single-photon sources (HSPSSs) such as those based on four-wave mixing [40, 41] and spontaneous parametric down-conversion (SPDC) [21, 24, 25, 27, 42–44].

Single-photon technologies are nowadays of the utmost relevance in several research branches, and even for upcoming commercial implementations (e.g. secure communication); this makes their characterization a crucial metrological task. The multi-photon component in SPSs emission is one of the key parameters of such devices, and so a standard methodology to properly quantify this parameter would be of major interest for a large scientific community.

In a joint effort to provide a robust procedure suited for this task, three metrological institutes (INRiM, NPL and PTB) have participated in a measurement campaign addressed to the measurement of the multi-photon emission of a particular prototype of SPD-based continuous wave (CW) HSPS [45, 46], characterized by a very low number of residual non-heralded ‘noise’ photons granting a small multi-photon output. This research aims to prepare a robust procedure for an international metrological comparison on the \( g^{(2)} \) measurements of extremely faint light CW sources (i.e. at the single photon level) in the telecom C-band.

An analogous effort has been addressed to establish a proper procedure for the measurement of the \( g^{(2)} \) function of a single-emitter pulsed SPS (based on nitrogen vacancies in diamonds emitting in the visible range); the related results can be found in [47]. These joint efforts will be crucial for the development of an international metrological infrastructure for the characterization of both CW and pulsed SPSs, paving the way to the commercial success of the forthcoming quantum photonics related technologies.

In the following, the results achieved for the CW telecom HSPS will be presented.

2. The single-photon source

In our experimental setup (figure 1) a CW laser (\( \lambda = 532 \text{ nm} \)) pumps a 10 mm \( \times \) 1 mm \( \times \) 10 mm periodically-poled lithium niobate (PPLN) crystal, producing non-degenerate SPDC. We choose signal and idler photons with wavelengths of \( \lambda_s = 1550 \text{ nm} \) and \( \lambda_i = 810 \text{ nm} \), respectively. The idler photon is sent to an interference filter (IF) with a full width at half maximum (FWHM) of 10 nm, then fiber-coupled and sent to a silicon single-photon avalanche detector (Si-SPAD), heralding the arrival of a 1550 nm signal photon. The signal photon is addressed to a 30 nm FWHM IF and the trigger events \( \Delta t \equiv \Delta \text{switch} = 7 \text{ ns} \) which corresponds to the passage of a 1550 nm photon, and then switches to channel B for a chosen minimum ‘sleep’ time \( t_{\text{dead}} \simeq 11 \mu \text{s} \) before accepting a new heralding. This way we can adjust the rate at which single photons are emitted from our device, granting a minimum time between subsequent photons, thus avoiding dead time issues with many detectors.

For the purpose of this joint measurement, the HSPS output is connected to a 50:50 fiber beam splitter (FBS) whose outputs are sent to two Hanbury Brown & Twiss (HBT) interferometers, one belonging to INRiM and the other to the guest NMI (PTB or NPL). This configuration allows the performance of simultaneous data collection between INRiM and the guest NMI, avoiding any mismatch between measurements because of some drift in the HSPS output over time. This way, we have two joint measurement sessions, one involving INRiM and NPL (session INRiM-NPL) and one with INRiM and PTB (session INRiM-PTB).

Every HBT involved is composed of two infrared InGaAs-InP SPADs, be they free-running or gated. In gated mode, the SPADs are triggered by the same FPGA signal that goes to the OS. In each HBT, the outputs of the two InGaAs SPADs are sent to time-tagging coincidence electronics.

3. The measurement

Looking at the temporal histogram of one of the INRiM HBT gated SPADs (figure 2), we can distinguish three different ‘regions’ corresponding to:

- \( N_{\text{Her}} \) = true heralded photon counts;
- \( N_{\text{Bkg}} \) = counts due to unheralded background photons passing through the optical switch;
- \( N_{\text{Dark}} \) = dark counts of the detector.

From these three contributions, it is straightforward to write the total number of counts of the detector as \( N_{\text{Tot}} = N_{\text{Her}} + N_{\text{Bkg}} + N_{\text{Dark}} \).

For the \( i \)th SPAD of each HBT \( (i = 1, 2) \), we define the total photon detection probability \( P_i^{(\text{Ph})} \) as the ratio between the photon count events \( N_i^{(\text{Ph})} \) and the trigger events \( N_i^{(\text{Trig})} \), i.e.:

\[
P_i^{(\text{Ph})} = N_i^{(\text{Ph})} / N_i^{(\text{Trig})} = \frac{N_i^{(\text{Her})} + N_i^{(\text{Bkg})}}{N_i^{(\text{Trig})}} = \frac{N_i^{(\text{Tot})} - N_i^{(\text{Dark})}}{N_i^{(\text{Trig})}}. \tag{1}
\]

Analogously, the dark count probability will be \( P_i^{(\text{Dark})} \equiv N_i^{(\text{Dark})} / N_i^{(\text{Trig})} \), and the overall detection probability will be \( P_i^{(\text{Tot})} = P_i^{(\text{Ph})} + P_i^{(\text{Dark})} \). By carefully tuning the ‘sleep’ time \( t_{\text{dead}} \) of the FPGA controlling the OS, in case of gated SPADs or dark-count-free detectors we can always have both detectors ready (i.e. out of their dead time) for each accepted heralding count, thus \( N_i^{(\text{Trig})} = N_i^{(\text{2Trig})} = N^{(\text{Trig})} \). With free-running
detectors, instead, \(N_{\text{Trig}}\) is just the number of heralding signals accepted by the FPGA (indicating the number of times the OS has been open), and a dead-time correction must be introduced (unless the dark counts are negligible).

Note that we have made the assumption that all events represented by \(P\)'s are mutually exclusive and independent of each other. This is a reasonable assumption in the situation where these probabilities are \(<1\), and the HBT SPADs settings are tuned in order to make afterpulses indistinguishable from dark counts. To evaluate the photon count and dark count probabilities, we look at the temporal histograms of the SPADs detection events (as in figure 2).

Actually, one might point out that, even with the three orders of magnitude extinction ratio \(r\) granted by the optical switch, the possibility of unwanted photons leaking through the OS while the HSPS shutter is closed is still present.
affecting the dark counts level estimation. Concerning this, we carefully evaluated the leaking photons contribution within the detection window and found it to be way below the statistical uncertainty associated to the dark counts. This means that the photon leakage effect in our HSPS is substantially negligible.

The following analysis allows us to use the time-tagged counts from individual detectors in combination with a time-correlated measurement from both to calculate Glauber’s normalized second-order autocorrelation function:

\[ g^{(2)}(\tau) = \lim_{t \to \infty} \frac{\langle I(t) I(t + \tau) \rangle}{\langle I(t) \rangle \langle I(t + \tau) \rangle}, \]

with \( I(t) \) being the photon emission intensity at time \( t \). For our purpose, the most interesting case appears at zero time difference (\( \tau = 0 \)), since an ideal SPS should present \( g^{(2)}(0) = 0 \), meaning that the source is producing anti-bunched single photons. Hence, the smaller the \( g^{(2)}(0) \), the better the performances in terms of single-photon emission. Actually, what we evaluate for our HSPS is the parameter \( \alpha \) (approximating the second order correlation function \( g^{(2)}(0) \)) for \( P_i^{\text{Tot}} \ll 1 \) [48]:

\[ \alpha = \frac{P_{12}^{\text{Tot}}}{P_{1}^{\text{Tot}} P_{2}^{\text{Tot}}} \approx g^{(2)}(0), \]

where \( P_{12}^{\text{Tot}} \) is the probability of a coincidence photon count between the two HBT SPADs (dark counts subtracted). Since \( P_{12}^{\text{HerHer}} \approx 0 \) (since we have a probability below \( 10^{-6} \) of getting more than one heralded photon within a 1 ns time interval, which is a generous estimate of the detector jitter time), and \( P_{1}^{\text{Bkg}} \) is independent from \( P_{2}^{\text{Bkg}} \), one has:

\[ P_{12}^{\text{Ph,Ph}} = P_{1}^{\text{Tot}} P_{2}^{\text{Tot}} - P_{12}^{\text{Dark}} - P_{1}^{\text{Dark}} P_{2}^{\text{Tot}} + P_{1}^{\text{Dark}} P_{2}^{\text{Dark}}. \]

For each HBT SPAD, the quantity \( N_i^{\text{Ph}} \) is extracted as the difference between the counts of the photon peak region (\( N_i^{\text{peak}} \)), integrated on a 10 ns time window and the counts belonging to a corresponding number of time bins in the dark count region (\( N_i^{\text{dark=peak}} \)), i.e., \( N_i^{\text{Ph}} = N_i^{\text{peak}} - N_i^{\text{dark=peak}} \). From this, we can compute \( N_i^{\text{Dark}} = N_i^{\text{Tot}} - N_i^{\text{Ph}} \), and then we have all we need to evaluate the single-count probabilities in equation (4). The total coincidence probability \( P_{12}^{\text{Tot}} \) is \( \frac{N_i^{\text{Conc}}}{N_i^{\text{Tot}}} \), being \( N_i^{\text{Conc}} \) the number of two-photon counts events in the HBT, independently evaluated by the coincidence electronics associated to it.

A further refinement for the \( \alpha \) parameter evaluation arises when looking at the multichannel picture in figure 2: it is in fact evident that the portion of dark counts on the left of the photon peak plays a different role with respect to the rest of them, since they occur before the arrival of the heralded photons. Indeed, this portion of dark counts forbids the SPAD to detect the heralded photons. This implies that such events (that we will indicate with \( N_i^{\text{Null}} \)) should be removed from the valid trigger events as well as from the SPAD counts, and thus the true valid trigger counts, dark counts and total counts will be, respectively: \( N_i^{\text{Trig}} = N_i^{\text{Trig}} - N_i^{\text{Null}} \), \( N_i^{\text{Dark}} = N_i^{\text{Dark}} - N_i^{\text{Null}} \), \( N_i^{\text{Tot}} = N_i^{\text{Tot}} - N_i^{\text{Null}} \).

This will affect all the different count probabilities, and thus the new formula for the \( \alpha \) parameter will be:

\[ \alpha = \frac{P_{12}^{\text{Tot}}}{P_{1}^{\text{Tot}} P_{2}^{\text{Tot}}} \cdot \frac{P_{12}^{\text{Tot}}}{P_{1}^{\text{Tot}} P_{2}^{\text{Tot}}} \]

(5)

By introducing the coefficient \( q_i = N_i^{\text{Trig}}/(N_i^{\text{Trig}} - N_i^{\text{Null}}) \), one can write \( P_{12}^{\text{Ph,Ph}} = q_i q_2 P_{12}^{\text{Tot}} \). Analogously, one can derive for the coincidence count probability the relation

\[ \alpha = \frac{P_{12}^{\text{Ph,Ph}}}{P_{1}^{\text{Ph}} P_{2}^{\text{Ph}}} = \frac{q_1 q_2 P_{12}^{\text{Tot}}}{P_{1}^{\text{Ph}} P_{2}^{\text{Ph}}} \]

(6)

that shows how such correction, even though sensible for both single and coincidence count probabilities, does not affect \( \alpha \).

4. The HBT interferometers

INRIM’s HBT comprises a 50:50 telecom FBS and two calibrated Micro Photon Devices single photon counters based on InGaAs/InP SPADs, operating in gated mode with a 30 ns detection window. The two SPADs outputs, together with the FPGA gating signal triggering them, are addressed to a time-tagging coincidence electronics (PicoQuant HydraHarp400) with a 2.5 ps time-bin resolution as well as to a time-to-amplitude converter (TAC) module, in order to have a separate evaluation of the single counts and coincidence counts of the HBT SPADs. Both SPADs detection windows being 30 ns long, we set a 100 ns coincidence window both in the time-tagging electronics and the TAC, to not miss any coincidence event.

Rearranging equation (3) gives:

\[ \alpha = \frac{P_{12}^{\text{Tot}}}{P_{1}^{\text{Tot}} P_{2}^{\text{Tot}}} \left( \frac{Q_1^{\text{Dark}} + Q_2^{\text{Dark}}}{Q_1^{\text{Trig}} + Q_2^{\text{Trig}}} \right) \times \left( \frac{Q_2^{\text{Dark}}}{Q_2^{\text{Trig}}} \right), \]

being \( P_{12}^{\text{Tot}} = \langle N_i^{\text{Trig}} \rangle / N_i^{\text{Trig}} \) and \( Q_i^{\text{Dark}} = N_i^{\text{Dark}} / N_i^{\text{Trig}} \), respectively, the total count probability and the fraction of dark counts registered by the \( i \)th SPAD of the HBT. The \( Q_i^{\text{Dark}} \) quantities are extracted by the multi-channel analyzer (MCA) histograms of the two detectors forming the HBT, integrating all the counts registered by the time-tagging coincidence electronics within the whole acquisition time. On the other hand, the \( P_{12}^{\text{Tot}} \) and \( P_{12}^{\text{Trig}} \) probabilities are obtained from the TAC counts, acquired in sets lasting 100 s each (the \( \langle N_i \rangle \) quantities are the mean values of these repeated acquisitions).

The NPL HBT hosts two different ID Quantique detectors, a gated SPAD (ID210, with a 25 ns detection window) and a low-noise free-running SPAD (ID230). The SPAD outputs, together with the FPGA valid gate output (i.e., the heralding counts validated by the FPGA and used for the HBT SPADs gating), are routed to the HydraHarp400 time-tagger. Since one of the SPADs is free-running, to have a reliable estimate
of the single- and two-photon count events we choose a 200 ns time window in the time-tagging electronics. A first 1000 s acquisition is run feeding the HydraHarp400 with the HBT SPADs output going to the HydraHarp 400 channels 1 and 2, respectively, while the FPGA valid gate output is used as reference external clock. Then, a second 2000 s run is instead performed with the ID210 as the external clock and the ID230 as channel 1 input. Finally a third 1000 s acquisition is collected with the same settings as the first one. This method gives the possibility of making a time-correlated evaluation of the HBT coincidence counts (during the 2000 s acquisition), whilst minimising the (eventual) temporal fluctuations and drifts of the photon source under test. Since one of the HBT detectors is free-running, we need to set a sensible collection time window on the HydraHarp400; to be sure not to loose any significant events, we choose a 200 ns collection time window both for single- and two-photon events acquisition.

Here, the total single-photon count probabilities will be
\[ P_i^{(Tot)} = \frac{N_i^{(Tot)}}{N_{NS}}, \]
with \( N_i^{(Tot)} \) the number of FPGA valid gates belonging to the single-photon event acquisitions, while the two-photon count probability will be
\[ P_{12}^{(Tot:Tot)} = \frac{N_i^{(Coinc)}}{N_{NS}}, \]
being \( N_i^{(Coinc)} \) the FPGA valid gates related to the two-photon events acquisition. The \( Q_i^{(Dark)} \) estimation remains instead unchanged.

The PTB HBT hosts two ID Quantique gated SPADs (one ID210 and one ID201, with, respectively, 25 ns and 50 ns detection windows), both triggered by the same FPGA valid gate signal. Since the time-tagging system chosen is a PicoQuant PicoHarp300, hosting only one input channel plus an external clock, the simultaneous evaluation of the single- and two-photon counts is not feasible. For this reason, we use the same method as the second INRIM-NPL measurement session, with a first 1000 s acquisition in which the single counts of both detectors at once were evaluated (by feeding the joined output signals of the HBT SPADs to the PicoHarp300 input channel, adding a proper time delay between the two to allow discrimination, and using the FPGA valid gates as external clock), a second 2000 s acquisition with the ID210 output as clock and the ID201 output as signal to estimate the two-photon counts, and a third 1000 s acquisition with the same settings as the first one. As for the INRIM HBT, since the sum of the two PTB SPADs detection windows is \( \approx 75 \text{ ns} \), for the two-photon coincidence events we set a 100 ns window in the time-tagging electronics.

Concerning the data analysis, we adopt the same method as the INRIM-NPL session for the evaluation of \( \alpha \) and the related uncertainty.

### 5. Results

Table 1 hosts the results obtained by INRIM, PTB and NPL during the different measurement sessions, namely, the concurrent measurements of the \( g^{(2)}(0) \) of the HSPS carried out firstly by INRIM and NPL and then by INRIM and PTB.

The uncertainty budgets related to the INRIM measurements are listed in tables 2 and 3, showing the single uncertainties, sensitivity coefficients and contributions to the total uncertainty on \( \alpha \) for each measurement session.

### Table 1. Experimental results obtained within the pilot comparison on the parameter \( \alpha \), with a coverage factor \( k = 1 \).

<table>
<thead>
<tr>
<th>Session</th>
<th>( \alpha ) (INRIM)</th>
<th>( \alpha ) (NPL)</th>
<th>( \alpha ) (PTB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INRIM-NPL</td>
<td>0.013 ± 0.008</td>
<td>0.02 ± 0.02</td>
<td>—</td>
</tr>
<tr>
<td>INRIM-PTB</td>
<td>0.016 ± 0.006</td>
<td>—</td>
<td>0.04 ± 0.05</td>
</tr>
</tbody>
</table>

### Table 2. INRIM uncertainty budget related to the INRIM-NPL measurement session. Coverage factor \( k = 1 \). ‘Unc.’: uncertainty. ‘Sens. Coeff.’: sensitivity coefficient. ‘Unc. Contr.’: contribution to the global uncertainty on \( \alpha \).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_1^{(Dark)} )</td>
<td>0.05604</td>
<td>0.00008</td>
<td>1.0461</td>
</tr>
<tr>
<td>( Q_2^{(Dark)} )</td>
<td>0.05607</td>
<td>0.00008</td>
<td>1.0462</td>
</tr>
<tr>
<td>( N_1^{(Trig)} )</td>
<td>6.0133 × 10^6</td>
<td>2.6 × 10^3</td>
<td>2.242 × 10^-8</td>
</tr>
<tr>
<td>( N_1^{(Tot)} )</td>
<td>18 261</td>
<td>31</td>
<td>7.382 × 10^-6</td>
</tr>
<tr>
<td>( N_2^{(Tot)} )</td>
<td>19 396</td>
<td>32</td>
<td>6.950 × 10^-6</td>
</tr>
<tr>
<td>( N_i^{(Coinc)} )</td>
<td>7.1</td>
<td>0.4</td>
<td>0.019 05</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.013</td>
<td>0.008</td>
<td>—</td>
</tr>
</tbody>
</table>

### Table 3. INRIM uncertainty budget related to the INRIM-PTB measurement session. Coverage factor \( k = 1 \). ‘Unc.’: uncertainty. ‘Sens. Coeff.’: sensitivity coefficient. ‘Unc. Contr.’: contribution to the global uncertainty on \( \alpha \).

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>( Q_1^{(Dark)} )</td>
<td>0.04525</td>
<td>0.00008</td>
<td>1.0302</td>
</tr>
<tr>
<td>( Q_2^{(Dark)} )</td>
<td>0.04875</td>
<td>0.00009</td>
<td>1.0340</td>
</tr>
<tr>
<td>( N_1^{(Trig)} )</td>
<td>6.1885 × 10^6</td>
<td>2.4 × 10^3</td>
<td>1.898 × 10^-8</td>
</tr>
<tr>
<td>( N_1^{(Tot)} )</td>
<td>22 490</td>
<td>41</td>
<td>5.223 × 10^-6</td>
</tr>
<tr>
<td>( N_2^{(Tot)} )</td>
<td>23 407</td>
<td>43</td>
<td>5.018 × 10^-6</td>
</tr>
<tr>
<td>( N_i^{(Coinc)} )</td>
<td>9.1</td>
<td>0.5</td>
<td>0.01294</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.016</td>
<td>0.006</td>
<td>—</td>
</tr>
</tbody>
</table>

### Table 4. Uncertainty budget related to the NPL measurement of the INRIM-NPL measurement session. Coverage factor \( k = 1 \). For the evaluation of the global uncertainty on \( \alpha \), we set \( \xi = 2 \). ‘Unc.’: uncertainty. ‘Sens. Coeff.’: sensitivity coefficient. ‘Unc. Contr.’: contribution to the global uncertainty on \( \alpha \).

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<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>( N_1^{(Trig)} )</td>
<td>1.20348 × 10^8</td>
<td>( \xi ) · 1.1 × 10^4</td>
<td>1.042 × 10^-9</td>
</tr>
<tr>
<td>( N_1^{(Tot)} )</td>
<td>304 900</td>
<td>( \xi ) · 600</td>
<td>1.726 × 10^-7</td>
</tr>
<tr>
<td>( N_2^{(Tot)} )</td>
<td>283 300</td>
<td>( \xi ) · 600</td>
<td>1.003 × 10^-7</td>
</tr>
<tr>
<td>( N_1^{(Dark)} )</td>
<td>3100</td>
<td>60</td>
<td>3.256 × 10^-6</td>
</tr>
<tr>
<td>( N_2^{(Dark)} )</td>
<td>9600</td>
<td>100</td>
<td>3.590 × 10^-6</td>
</tr>
<tr>
<td>( N_i^{(Coinc)} )</td>
<td>1.20184 × 10^8</td>
<td>( \xi ) · 1.1 × 10^4</td>
<td>5.218 × 10^-10</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.02</td>
<td>0.02</td>
<td>—</td>
</tr>
</tbody>
</table>
Table 5. Uncertainty budget related to the PTB measurement of the INRiM-PTB measurement session. Coverage factor \( k = 1 \). For the evaluation of the global uncertainty on \( \alpha \), we set \( \xi = 2 \). ‘Unc.:’ uncertainty. ‘Sens. Coeff.:’ sensitivity coefficient. ‘Unc. Contr.:’ contribution to the global uncertainty on \( \alpha \).

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>( N_i^{(\text{Tog})} )</td>
<td>( 1.23807 \times 10^8 )</td>
<td>( \xi \cdot 1.1 \times 10^4 )</td>
<td>( 1.031 \times 10^{-6} )</td>
</tr>
<tr>
<td>( N_i^{(\text{Tot})} )</td>
<td>453 500</td>
<td>( \xi \cdot 700 )</td>
<td>1.140 \times 10^{-6}</td>
</tr>
<tr>
<td>( N_i^{(\text{Trig})} )</td>
<td>474 200</td>
<td>( \xi \cdot 700 )</td>
<td>4.884 \times 10^{-7}</td>
</tr>
<tr>
<td>( N_i^{(\text{Dark})} )</td>
<td>50 700</td>
<td>300</td>
<td>2.390 \times 10^{-6}</td>
</tr>
<tr>
<td>( N_i^{(\text{Coinc})} )</td>
<td>140 800</td>
<td>400</td>
<td>2.888 \times 10^{-6}</td>
</tr>
<tr>
<td>( N_i^{(\text{Loc})} )</td>
<td>1.23733 \times 10^8</td>
<td>( \xi \cdot 30 )</td>
<td>5.158 \times 10^{-9}</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.04</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

The uncertainty budget of the INRiM-NPL and INRiM-PTB measurement sessions are instead given in tables 4 and 5, respectively. Since here we do not have repeated measurements providing information on the photon counts fluctuations (in our regime, dark counts obey a Poisson distribution), we give an upper bound for them considering a superpoissonian behavior by putting \( u(N) = \xi \sqrt{N} \), with \( \xi > 1 \).

6. Conclusions

We have illustrated a strategy to properly evaluate the multi-photon component of a CW light source, applied to a low-noise prototype of a fiber HSPS @1550 nm particularly suited for metrological and quantum-communication-related purposes and adaptable to a large variety of detectors and other devices. The results of the whole measurement campaign, carried out with different measurement setups and data collection methodologies, are all in agreement within the experimental uncertainties, even with coverage factor \( k = 1 \).

The proposed strategy may pave the way to a standardization of the characterisation of single-photon sources, a task of the utmost relevance for present and future metrology for quantum technologies.

Acknowledgments

This work has been supported by EMPIR-14IND05 ‘MIQ2C’, EMPIR-17FUN06 ‘SIQUEST’ and EMPIR-17FUN01 ‘BECOME’ (the EMPIR initiative is co-funded by the EU H2020 and the EMPIR Participating States).

Appendix. Uncertainties evaluation

A.1. INRiM uncertainty budgets

For what concerns the uncertainty budget related to INRiM measurements, we can identify three different contributions:

- \( u(P_i^{\text{12}}) \): uncertainty derived from the double (\( P_i^{\text{Tog,Tot}} \)) and single (\( P_i^{\text{Tot}} \)) photon count probabilities, given by the TAC counts;
- \( u(Q_i^{\text{Dark}}) \), \( i = 1, 2 \): uncertainty derived from the \( Q_i^{\text{Dark}} \) fractions of each SPAD of the HBT, evaluated from the HydraHarap400 histograms.

The first uncertainty contribution, i.e. \( u(P_i^{\text{12}}) \), is evaluated in the following way:

\[
\begin{align*}
\quad u(P_i^{\text{12}}) & = \sum_{m=0}^{3} C_{lm} \left( \frac{\partial u}{\partial N_l} \right) \left( \frac{\partial u}{\partial N_m} \right) u(N_l) u(N_m) \\
\quad C_{lm} & = \frac{(\langle N_i(N_m) \rangle - \langle N_i \rangle \langle N_m \rangle)}{u(N_i) u(N_m)}
\end{align*}
\]

being \( \langle N_i \rangle \), \( \langle N_m \rangle \) the standard deviation on the average \( \langle N_i \rangle \). Table A1 hosts the correlation coefficients \( C_{ij} \) (\( i, j = 0, 1, 2 \ldots 3 \)) among the \( N_i \)'s for all of the INRiM measurement sessions.

Concerning the uncertainties contributions \( u(Q_i^{\text{Dark}}) \), we have to consider that extracting them just from the HydraHarap400 histograms would lead to an overestimation of the total uncertainty \( u(\alpha) \); on the contrary, knowing that \( Q_i^{\text{Dark}} = \frac{N_i^{\text{Dark}}}{N_i^{\text{Em}}EMPIR} - 1 = \frac{\tilde{N}_i^{\text{Dark}}}{\tilde{N}_i^{\text{Em}}EMPIR} \), we can write:

\[
\begin{align*}
\quad u^2(Q_i^{\text{Dark}}) & = \left( \frac{1}{\tilde{N}_i^{\text{Em}}EMPIR} \right)^2 u^2(\tilde{N}_i^{\text{Ph}}) + \left( \frac{\tilde{N}_i^{\text{Ph}}}{\tilde{N}_i^{\text{Em}}EMPIR} \right)^2 u^2(\tilde{N}_i^{\text{Tot}}) \\
\quad & - 2 \left( \frac{1}{\tilde{N}_i^{\text{Em}}EMPIR} \right) \left( \frac{\tilde{N}_i^{\text{Ph}}}{\tilde{N}_i^{\text{Em}}EMPIR} \right) (\langle \tilde{N}_i^{\text{Tot}} \rangle \langle \tilde{N}_i^{\text{Ph}} \rangle - \langle \tilde{N}_i^{\text{Tot}} \rangle \langle \tilde{N}_i^{\text{Ph}} \rangle)
\end{align*}
\]

Since \( \tilde{N}_i^{\text{Em}}EMPIR = \tilde{N}_i^{\text{Ph}} + \tilde{N}_i^{\text{Dark}} \), with some algebraic passages one has:

\[
\quad (\langle \tilde{N}_i^{\text{Tot}} \rangle \tilde{N}_i^{\text{Ph}} - \langle \tilde{N}_i^{\text{Tot}} \rangle \langle \tilde{N}_i^{\text{Ph}} \rangle) = u^2(\tilde{N}_i^{\text{Ph}}) \leq u^2(\tilde{N}_i^{\text{Tot}}),
\]

and hence:

\[
\quad u(Q_i^{\text{Dark}}) \leq \frac{Q_i^{\text{Dark}}}{\tilde{N}_i^{\text{Tot}}} u(\tilde{N}_i^{\text{Tot}}),
\]

giving a sensible upper bound on the uncertainty contributions \( u(Q_i^{\text{Dark}}) \).

Table A1. Correlation coefficients related to the INRiM measurements in both sessions.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>INRiM-NPL</th>
<th>INRiM-PTB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{0,1} )</td>
<td>0.326</td>
<td>0.857</td>
</tr>
<tr>
<td>( C_{0,2} )</td>
<td>0.445</td>
<td>0.891</td>
</tr>
<tr>
<td>( C_{0,3} )</td>
<td>0.269</td>
<td>0.133</td>
</tr>
<tr>
<td>( C_{1,2} )</td>
<td>0.650</td>
<td>0.824</td>
</tr>
<tr>
<td>( C_{1,3} )</td>
<td>0.002 61</td>
<td>-0.000 155</td>
</tr>
<tr>
<td>( C_{2,3} )</td>
<td>-0.000 909</td>
<td>0.001 61</td>
</tr>
</tbody>
</table>

\( u(\alpha) \)
\[ u(\alpha) = \sqrt{u^2(\alpha) + u^2_{(Q_2)}(\alpha) + u^2_{(P)}(\alpha)}. \] (A.5)

### A.2. NPL and PTB uncertainty budgets

Concerning NPL and PTB measurements, for evaluating the uncertainty \( u(\alpha) \) we use:

\[ u(\alpha) = \sqrt{\sum_{i=0}^{N_0} \left( \frac{\partial \alpha}{\partial N_i} \right)^2 u^2(N_i)}, \] (A.6)

with \( N_0 = N^{(\text{Trig})} \), \( N_i = N^{(\text{Tot})}_i \) and \( N_{2,1} = N^{(\text{Dark})}_i \) \((i = 1, 2)\), \( N_4 = N^{(\text{C})} \) and, finally, \( N_6 = N^{(\text{conc})} \).

Since here we do not have repeated measurements, we provide a sensible upper bound to the measurement uncertainties by statistical considerations on the physics of the source and measurement device. The dark counts of the HBT SPADs are known to follow a Poisson distribution. The statistics of a multi-mode SPDC, like the one exploited in our experiment, is almost indistinguishable from a Poissonian distribution. For these reasons, we can assume the fluctuations of the \( N_i \) quantities in (A.6) to be Poissonian as well, and choose as a coefficient to be tuned taking into account the (eventual) superpoissonian behavior of the system. In our case, having observed some unexpected fluctuations in the CW pump power, to be conservative we choose \( \xi = 2 \).

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